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## GEOMETRY.

**489. Proposed by NATHAN ALTSHILLER, The University of Colorado.**

The parallels to the asymptotes  $a, b$  of a given hyperbola, drawn from a variable point of the curve, meet  $a$  and  $b$  in  $P, Q$  respectively. The line  $PQ$  envelops an hyperbola whose asymptotes are  $a$  and  $b$ .

**490. Proposed by ELMER E. MOOTS, University of Arizona.**

In any quadrilateral  $ABCD$ , let  $AC$  and  $BD$  be the diagonals intersecting in  $K$ . On  $AC$ , lay off  $CR$  equal to  $AK$ . Join  $B$  and  $R$ . Connect the middle point  $G$  of  $BR$  with  $D$ . On  $GD$ , lay off  $GM$  equal to  $\frac{1}{3}GD$ . Show that  $M$  is the center of gravity of the quadrilateral.

**491. Proposed by N. P. PANDYA, Sojitra, India.**

In a triangle  $mx = b$  and  $nx = c$ , determine a relation between  $m, n, x, A$  and  $s$  and solve it for  $x$ .

## CALCULUS.

**407. Proposed by PAUL CAPRON, Annapolis, Maryland.**

A coffee pot in the form of a conical frustum, 10 inches high, with a lower base 8 inches in diameter and an upper base 6 inches in diameter, is held on a slant so that the lower base is barely covered by the coffee within, and the upper base is barely uncovered. How much coffee does the pot contain?

**408. Proposed by CLIFFORD N. MILLS, Brookings, South Dakota.**

The ellipse  $(x^2/81) + (y^2/16) = 1$  is revolved around the  $y$ -axis. Find the area of the surface generated.

**409. Proposed by B. J. BROWN, Victor, Colorado.**

Integrate the equation

$$\frac{\partial^2 z}{\partial x \partial y} + \frac{1}{x+y} \left( \frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} \right) - \frac{2}{(x+y)^2} z = 0.$$

## MECHANICS.

**326. Proposed by CLIFFORD N. MILLS, Brookings, South Dakota.**

A uniform beam, of length  $2l$ , rests in equilibrium against a smooth vertical wall and upon a peg at a distance  $a$  from the wall; show that the inclination of the beam to the vertical is

$$\sin^{-1} \left( \frac{a}{l} \right)^{\frac{1}{3}}.$$

**327. Proposed by C. N. SCHMALL, New York City.**

An inclined plane makes an angle  $\phi$  with the horizontal plane, and from its foot a body is projected upward at an angle  $\psi$  to the plane, and with velocity  $v$ . Show that it will strike the plane *perpendicularly* if  $\tan \psi = \frac{1}{3} \cot \phi$  and that its range up the plane in that case will be

$$\frac{2v^2 \sin \phi}{g(1 + 3 \sin^2 \phi)}.$$

## NUMBER THEORY.

**244. Proposed by CLIFFORD N. MILLS, Brookings, South Dakota.**

Determine the rational value of  $x$  that will render  $x^2 + px + q$  a perfect square. What value of  $x$  will render  $x^2 - 7x + 2$  a perfect square?

**245. Proposed by NORMAN ANNING, Chilliwack, B. C.**

When all the letters denote positive integers and when the  $a$ 's are primes of the form  $4k + 1$ , the equation

$$x^2 + y^2 = (a_1 a_2 a_3 \cdots a_m)^n,$$

has, in Legendre's notation,  $E((n+1)^m/2)$  solutions. Show that in  $2^{m-1}$  of these solutions  $x$  and  $y$  are relatively prime.

### SOLUTIONS OF PROBLEMS.

#### ALGEBRA.

#### 445. Proposed by S. A. JOFFE, New York City.

Sum the series

$$\binom{cn}{a} - \binom{a}{1} \binom{cn-c}{a} + \binom{a}{2} \binom{cn-2c}{a} - \cdots + (-1)^a \binom{cn-ca}{a}.$$

#### I. SOLUTION BY THE PROPOSER.

This is a generalization of the series forming the first member of equation (2) in the Proposer's solution of problem No. 424 (*June 1915 issue*, p. 205), obtained by multiplying the upper indices  $n, n-1, n-2, \dots$  by the constant factor  $c$ . Following the method employed in that solution, we find that the given series equals

$$\Delta^a \binom{cn-ca}{a},$$

the finite differences being taken with respect to  $n$ .

Now

$$\Delta_x \binom{cx}{a} = \binom{cx+1}{a} - \binom{cx}{a},$$

the second member of which may be written in the following form:

$$\begin{aligned} \binom{cx+c}{a} - \binom{cx+c-1}{a} + \binom{cx+c-1}{a} - \binom{cx+c-2}{a} + \binom{cx+c-2}{a} - \cdots \\ + \binom{cx+1}{a} - \binom{cx}{a}, \end{aligned}$$

since all these terms, except the first and last, alternately cancel each other. Combining the terms in pairs and noticing that

$$\binom{cx+c}{a} - \binom{cx+c-1}{a} = \binom{cx+c-1}{a-1}, \quad \binom{cx+c-1}{a} - \binom{cx+c-2}{a} = \binom{cx+c-2}{a-1},$$

etc., we have

$$\Delta_x \binom{cx}{a} = \binom{cx+c-1}{a-1} + \binom{cx+c-2}{a-1} + \cdots + \binom{cx+1}{a-1} + \binom{cx}{a-1},$$

which means that the first difference, taken with respect to  $x$ , of the binomial coefficient  $\binom{cx}{a}$  having for its lower index  $a$ , equals the sum of  $c$  binomial coefficients, each having for its lower index  $a-1$ .

In the same manner, the second difference  $\Delta_x^2 \binom{cx}{a}$  may be expressed as the sum of  $c \cdot c = c^2$  binomial coefficients, each having  $a-2$  for its lower index; and continuing this process, we find that the  $a$ th difference  $\Delta_x^a \binom{cx}{a}$  may be expressed as the sum of  $c^a$  binomial coefficients, each having for its lower index  $a-a=0$  and hence each equal to 1. In other words,

$$\Delta_x^a \binom{cx}{a} = c^a,$$

and, similarly,

$$\Delta_n^a \binom{cn-a}{a} = c^a.$$